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# Technical Comments

## Comment on "Performance Limits for Projectile Flight in the Ram and External Propulsion Accelerators"

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### Nomenclature

$a$ = acceleration
$C_D$ = drag coefficient
$D$ = drag
$d$ = projectile diameter
$M$ = Mach number
$m$ = projectile mass
$p$ = pressure
$Q$ = chemical energy release per unit mass
$S$ = projectile cross-sectional area, $\pi d^2/4$
$T$ = thrust
$V$ = projectile velocity
$W$ = projectile weight, mg
$\beta$ = ratio of combustion region diameter to projectile diameter
$\gamma$ = ratio of specific heats
$\eta_p$ = propulsive efficiency
$\rho$ = density

THE recent paper by Rom[1] attempts to evaluate the thrust and drag coefficient of a ram accelerator projectile using an energy balance analysis. Unfortunately, an error in the analysis invalidates the results of that paper. In this Technical Comment, the correct analysis will be derived, the prior paper's mistakes identified, and the overall merit of such an energy balance analysis assessed.

The author begins his analysis by defining a propulsive efficiency $\eta_p$

$$\eta_p = T/Q\rho_\infty\beta^2 S \tag{1}$$

and a drag coefficient

$$C_D = D/\tfrac{1}{2}\rho_\infty V_\infty^2 S \tag{2}$$

Thus, the thrust and drag on the projectile have the functional dependence $T = T(Q, \rho_\infty, \beta, S, \eta_p)$ and $D = D(\rho_\infty, V_\infty, S, C_D)$, where $\eta_p$ and $C_D$ are functions of projectile velocity and the details of the flowfield. In the paper under discussion (Ref. 1), the author uses experimental ram accelerator data in an attempt to determine the $\eta_p$ and $C_D$ as a function of projectile velocity. Using the experimentally measured acceleration at a given velocity

$$a = [T(Q, \rho_\infty, \beta, S, \eta_p) - D(\rho_\infty, V_\infty, S, C_D)]/m \tag{3}$$

and the velocity at which the projectile reaches a constant velocity and thrust equals drag

$$T(Q, \rho_\infty, \beta, S, \eta_{p_{\max}}) = D(\rho_\infty, V_{\max}, S, C_{D_{\max}}) \tag{4}$$

the author produces plots of $\eta_p$ and $C_D$ vs the distance the projectile travels along the ram accelerator tube. (Here, $\eta_{p_{\max}}$ and $C_{D_{\max}}$ denote the values of propulsive efficiency and drag coefficient at the maximum projectile velocity, respectively, not the maximum values of $\eta_p$ and $C_D$.) Equations (3) and (4), however, do not constitute a closed set sufficient to determine $\eta_p$ and $C_D$. Specifically, there are four unknowns ($\eta_p$, $C_D$, $\eta_{p_{\max}}$, and $C_{D_{\max}}$), and only two equations.

To identify the error that allows Rom to determine four unknowns from only two equations, it is necessary to rederive much of the analysis in the paper. The author begins by considering the static case when the thrust on the projectile exactly equals the drag, resulting in the maximum possible projectile velocity. From Eq. (4) this can be written

$$\rho_\infty\left[\frac{\pi(\beta d)^2}{4}\right]Q\eta_{p_{\max}} = \frac{1}{2}\rho_\infty V_{\max}^2\left(\frac{\pi d^2}{4}\right)C_{D_{\max}} \tag{5}$$

This equation is in agreement with Eq. (8) from Ref. 1, except for the addition of the max notation to $\eta_p$ and $C_D$. This notation is necessary because $\eta_p$ and $C_D$ are functions of velocity, and this equation only applies at the maximum projectile velocity.

Next, the author turns to the case of an accelerating projectile. From Eq. (3), the acceleration can be written as

$$\rho_\infty\left[\frac{\pi(\beta d)^2}{4}\right]Q\eta_p - \frac{1}{2}\rho_\infty V_\infty^2\left(\frac{\pi d^2}{4}\right)C_D = T - D = \frac{a}{g}W \tag{6}$$

which agrees with Eq. (11) in Ref. 1. Note that now $\eta_p$ and $C_D$ are variables whose values change with velocity. Using Eq. (5), this can be expressed as

$$\frac{a}{g} = \frac{\gamma M_\infty^2 C_D}{2}\left[\left(\frac{C_{D_{\max}}}{C_D}\right)\left(\frac{\eta_p}{\eta_{p_{\max}}}\right)\left(\frac{V_{\max}}{V_\infty}\right)^2 - 1\right]\frac{p_\infty S}{W} \tag{7}$$

Note the difference between this result and that of Eq. (12) in Ref. 1. The author's failure to differentiate between the instantaneous values of $\eta_p$ and $C_D$ and their values at maximum projectile velocity, $\eta_{p_{\max}}$ and $C_{D_{\max}}$, results in an erroneous simplification to the form

$$\frac{a}{g} = \frac{\gamma M_\infty^2 C_D}{2}\left[\left(\frac{V_{\max}}{V_\infty}\right)^2 - 1\right]\frac{p_\infty S}{W} \tag{8}$$

For Eqs. (7) and (8) to be equal, the ratio $\eta_p/C_D$ must be constant. This assumption cannot be made a priori because the stated purpose of Ref. 1 is to determine the independent variation of $\eta_p$ and $C_D$ from experimental data.

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By invoking Eq. (8), the author proceeds to use experimental measurements of projectile acceleration and maximum velocity to plot $\eta_p$ and $C_D$ as functions of projectile travel through the ram accelerator. As has been shown, Eq. (8) is a mistaken simplification of Eq. (7), and the results of using it are incorrect. The erroneous results of this analysis are clearly visible in Fig. 7 of Ref. 1, where the drag coefficient of a sharp-nosed projectile in supersonic flight is shown to approach values of $C_D = 1.5$. Of course, even a blunt-nosed projectile (such as a sphere) does not have such a high drag coefficient, and direct measurements of ram accelerator projectiles decelerating in inert gas have verified a drag coefficient ($C_D \sim 0.3$) similar to what a sharp-nosed projectile experiences in free flight.[2]

Finally, the overall value of such an energy balance analysis is questionable. As has been shown here, experimental measurement of acceleration and maximum velocity does not provide sufficient information to solve for the propulsive efficiency and drag coefficient. While the energy balance concept may have some merit for simple, order-of-magnitude estimates, the value of propulsive efficiency is highly coupled to the details of the particular flowfield around the projectile. Comparing the ram and external propulsion accelerators (which reply on entirely different mechanisms to initiate, stabilize, and extract thrust from the combustion process) on this basis is likely to lead to conclusions of dubious quality.

## References

[1]Rom, J., "Performance Limits for Projectile Flight in the Ram and External Propulsion Accelerators," *Journal of Propulsion and Power*, Vol. 13, No. 5, 1997, pp. 583–591.

[2]Knowlen, C., Higgins, A. J., Bruckner, A. P., and Bauer, P., "Ram Accelerator Operation in the Superdetonative Velocity Regime," AIAA Paper 96-0098, Jan. 1996.